Calculation of the Turning Angle of Two-Dimensional Incompressible Cascade Flow

Wolfgang Geller*
DFVLR, Porz Wahn, West Germany

Nomenclature

t = tangential spacing between blades
l = chord length
λ = staggering angle (0 for unstaggered cascades, $\lambda+\beta_1+\alpha_s=90°$)
w_1 = inlet velocity
β_1 = inlet angle
α_s = angle of attack (angle between inlet velocity and chord)
w_2 = outlet velocity of real flow
β_2 = outlet angle of real flow
$\Theta=\beta_2-\beta_1$ = turning angle
$(x_u/l)_s$ = distance between leading edge and transition point on the upper surface, measured on chord, referred to chord length 1
c_D = drag coefficient (total drag, referred to up-stream dynamic pressure and chord length)
$\Delta\beta_1$ = correction value for inlet angle, inserted into calculation
S = pressure coefficient (difference between up-stream total pressure and local pressure on blade surface, referred to up-stream dynamic pressure)
x/l = distance between control-point and leading edge, measured on chord, referred to chord length
d/l = ratio of maximum thickness and chord length
$Re=w_1\cdot l/\nu$ = Reynolds number
ν = kinematic viscosity

Theme

CALCULATING the inviscid subsonic flow past isolated profiles or aerofoils of cascades with cusped trailing edges, the circulation around a profile is clearly determined by the Kutta-Joukowsky condition. Blades of highly loaded turbomachines, however, have more or less thick generally rounded-off trailing edges. For these profiles the circulation and hence the flow deflection remains undetermined in a calculation according to potential flow theory, because the Kutta-Joukowsky condition cannot be applied. On the other hand, the turning angle is an experimental measurable item and has a reproducible value. It depends on the flow within the boundary layer, and especially on the location of the boundary-layer separation region at the trailing edge. Flow separation in the region of rounded-off trailing edges occurs in any case. The flow in the boundary layer is not able to overcome the pressure rise up to the value of the upstream total pressure, as would occur in the rear stagnation point of a body in potential flow. At two points, one on the upper and one on the lower surface of the profile, at least in the region of the trailing edge, the flow separates from the surface due to this pressure rise. A separated wake extending downstream is produced, which exhibits a region of approximately constant static pressure, at least in the vicinity of the profile. The wake interacts with the main flow, thus changing the pressure distribution in the whole flowfield, leaving invalid the data calculated by means of the potential flow theory without taking into account this interaction process.

Obviously, in order to obtain accurate theoretical results, one has to simulate in the calculation the displacement effect of the separated wake on the potential flow. As will be pointed out later, the circulation and hence the turning angle can be determined uniquely.

The foundations of the theory of separated flows were laid by H. Helmholtz,[1] and G. Kirchhoff.[2] They replaced the wake by a stagnant liquid, bounded towards the outer potential flow by discontinuity sheets. Both authors applied the hodograph method for the calculation of the flow.

For practical computing, however, it is more convenient to substitute the displacement effect of the actual separated wake by the displacement effect of a liquid, which is being discharged from sources located at the profile surface in the region of separation. The distribution of source strength per unit length may then be determined in such a manner that the static pressure remains constant at the profile surface in the whole separation region including both separation points.†

Contents

The present method for calculating the flow through cascades with separation is an extension of the well-known Martensen method.[4] Two assumptions are made 1) that the effect of the boundary-layer displacement on the pressure distribution can be neglected, and 2) that the simulated wake has an infinite length. It was found that these assumptions are without essential influence on the results.

The contours of the blade sections are replaced by vortex sheets. Source distributions are placed on the blade surfaces in the separation region between the separation point on the lower surface and the separation point on the upper surface. The fact that the tangential velocity at any point on the inner edge of the vortex sheet has to be equal to zero, gives a linear integral equation, which expresses the distribution of vortex density in terms of the velocity components tangential to the blade contours induced by the vortex and source distributions, and of the tangential component of the velocity of the undisturbed uniform flow. As can be shown by application of Stokes's and Gauss's theorems to a closed circuit, which includes an infinitesimal length of a vortex and source sheet, the vortex density is identical with the tangential velocity, and the source density identical with the normal velocity on the outer edge of the sheet.

The condition of constant pressure, being equivalent to the condition that the geometric sum of tangential and normal velocity has to be constant on the surface in the region of separation, gives a quadratic equation, which determines the source density in connection with the linear integral equation

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*Scientist, Institut für Angewandte Gasdynamik.

†Simulation of separated flow by a source distribution for the calculation of the pressure distribution on isolated aerofoils was also used by K. Jacob.[3] Jacob, however, satisfies the condition of constant pressure in three points only. These points are both the separation points and that point on the upper boundary of the simulated separated wake, which lies opposite to the trailing edge.

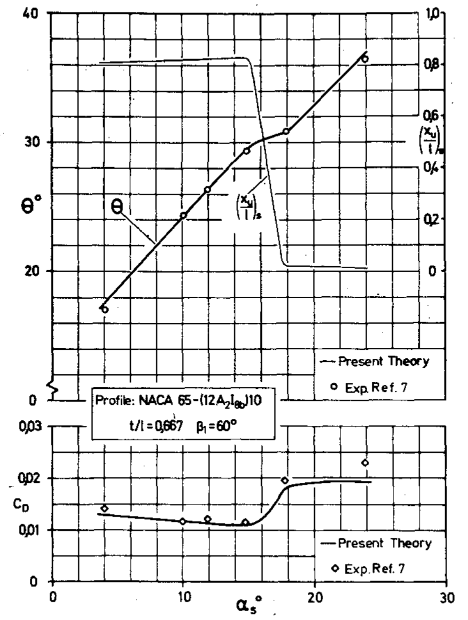

Fig. 1 Comparison of measured and calculated turning angles Θ and drag coefficients c_D for cascades of compressor blade sections NACA 65-($12A_2I_{8b}$) 10.Re.440,000.

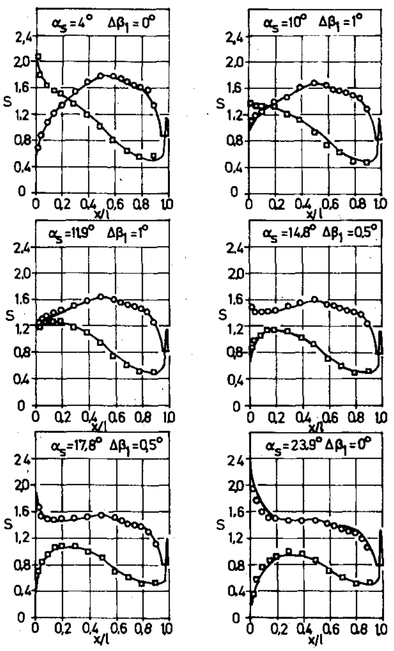

Fig. 2 Comparison of measured and calculated pressure distributions for the cascades of Fig. 1. ○ Measurement, Ref. 7 (upper surface); □ Measurement, Ref. 7 (lower surface); —present theory.

for the vortex distribution. Since the displacement effect of the boundary layer on the pressure distribution is being neglected, the source density in both separation points is equal to zero and one finds that the magnitude of the tangential velocities in these points have to be identical. By this condition the circulation of potential flow is specified, as previously mentioned.

Boundary-layer calculations in order to find the separation points were made using a method of Walz.[5] The position of both the separation points has to be estimated before starting the computation. The actual position is found by an iterative process employing alternatingly the potential flow theory and the boundary-layer method.

By application of the conservation laws of mass and momentum the turning angle of the flow, and the drag coefficient of the profile is calculated taking into account the displacement and momentum thickness of the boundary layer in both separation points (and only there) and the total source strength on the blade, respectively. A detailed description of the method is given in Ref. 6.

In Figs. 1 and 2 some typical results are given. Figure 1 shows calculated turning angles Θ and drag coefficients c_D, plotted vs angle-of-attack α_s, compared with results of NACA cascade tests on compressor blade sections.

The blade section was a NACA laminar profile of the 65 series, having a maximum thickness of 10%, and a maximum camber of about 8% of chord length. For all examples presented herein, it was assumed that the transition point is located in the point of laminar separation. In the upper half of the diagram the position of this point $(x_u/l)_s$ on the upper surface is shown. The results are strongly influenced by the location of upperside transition point only. The differences between measured and calculated turning angles are within the accuracy of measurement for the whole range of angles of attack. The agreement is very good. The change in gradient of the Θ-curve and the rapid increase of the drag coefficient at $\alpha_s = 15°$ agrees well with experimental results. This change is produced by the shifting of the separation point on the upper surface, which is due to a strong change of the location of the transition point towards the leading edge for larger angles of attack.

Figure 2 shows a comparison of measured and calculated blade-surface pressure distributions for the same cases. Instead of using the original inlet angle β_1, given in Ref. 7, slightly corrected values were substituted into the calculation. The correction values $\Delta\beta_1$, which are listed together with the angle-of-attack α_s in the upper box of each diagram, were determined in Ref. 8. There it was shown that, when using Martensen's method, a good agreement of measured with calculated pressure distributions could be achieved by inserting only the measured amount of circulation into the calculation. The solution could be improved further by a slight correction of the inlet angle. The correction values generally are less than 1°‡. It is conjectured that the difference between the given and the actual value of the inlet angle β_1 is due to effects of the boundary-layer control in the cascade tunnel by means of porous side walls.

The agreement of the measured and calculated pressure distributions is good. Similarly good results were obtained for other cascades. Obviously, the present method appears to be a suitable tool for the accurate prediction of the blade pressure distribution, the turning angle, and for the good estimation of the profile drag coefficient.

References

[1] Helmholtz, H., "Über diskontinierliche Flüssigkeitsbewegungen," KGL.Ak.d.Wiss., Berlin, Vol. 23, 1868, p. 215.

[2] Kirchhoff, G., "Zur Theorie freier Flüssigkeitsstrahlen," *J. reine angew. Math.*, Vol. 70, 1869, p. 289.

[3] Jacob, K., "Berechnung der abgelösten inkompressiblen Strömung um Tragflügelprofile und Bestimmung des maximalen Auftriebs," *Z. Flugwiss.*, Vol. 17, 1969, 7, pp. 221-230.

[4] Martensen, E., "Berechnung der Druckverteilung an Gitterprofilen in ebener Potentialströmung mit einer Fredholmschen Integral-gleichung," *Arch. Rat. Mech. Anal.*, Vol. 3, 1959, 3, pp. 235-270.

[5] Walz, A., "Strömungs- und Temperaturgrenzschichten," Karlsruhe: Braun, 1966.

[6] Geller, W., "Berechnung der Druckverteilung an Gitterprofilen in ebener inkompressibler Strömung mit Grenzschichtablösung im Bereich der Profilenden," DLR-Forschungsbericht 72-62.

[7] Erwin, J. R., Savage, M., and Emery, J. C., "Two-dimensional Low-Speed Cascade Investigation of NACA Compressor Blade Sections Having a Systematic Variation in Mean-Line Loading," NACA TN 3817, 1956.

[8] Geller, W., "Zur Berechnung der ebenen reibungsfreien Gitterströmung mit einem modifizierten Martensenverfahren," DLR-Forschungsbericht 68-77, 1968.

‡In the presentation of results, the turning angles Θ and the angles of attack are referred to the inlet angle β_1, as given by NACA, Ref. 7.